APPENDIX B
Mathematical Analysis of Underseepage and Substratum Pressure

B-1. General

The design of seepage control measures for levees and dams often requires an underseepage analysis without the use of piezometric data and seepage measurements. Contained within this appendix are equations by which an estimate of seepage flow and substratum pressures can be made, provided soil conditions at the site are reasonably well defined. The equations contained herein were developed during a study by the US Army Engineer Waterways Experiment Station (1956)[1] of piezometric data and seepage measurements along the Lower Mississippi River and confirmed by model studies. The following discussion is presented in terms of levee underseepage; however the analyses and equations are considered equally applicable to dam foundations. It should be emphasized that the accuracy obtained from the use of equations is dependent upon the applicability of the equation to the condition being analyzed, the uniformity of soil conditions, and the evaluation of the various factors involved. As is normally the case, sound engineering judgment must be exercised in determining soil profiles and soil input parameters for these analyses.

B-2. Assumptions

It is necessary to make certain simplifying assumptions before making any theoretical seepage analysis. The following is a list of such assumptions and criteria necessary to the analysis set forth in this appendix:

a. Seepage may enter the pervious substratum at any point in the foreshore (usually at riverside borrow pits) and/or through the riverside top stratum.

b. Flow through the top stratum is vertical.

c. Flow through the pervious substratum is horizontal. Flow in the vertical direction is entirely disregarded.

d. The levee (including impervious or thick berm) and the portion of the top stratum beneath it are impervious.

e. All seepage is laminar.

In addition to the above, it is also required that the foundation be generalized into a pervious sand or gravel stratum with a uniform thickness and permeability and a semipervious or impervious top stratum with a uniform thickness and permeability (although the thickness and permeability of the riverside and landside top stratum may be different).

B-3. Factors Involved in Seepage Analyses

The volume of seepage (Q_s) that will pass beneath a levee and the artesian pressure that can develop under and landward of a levee during a sustained high water are related to the basic factors given and defined in Table B-1 and shown graphically in Figure B-1. Other terms used in the analyses are defined as they are discussed in subsequent paragraphs.

B-4. Determination of Factors Involved in Seepage Analyses

Many of the factors necessary to perform a seepage analysis, such as exploration and testing, have previously been mentioned in the text; however they are discussed in more detail as they apply to each specific factor. The use of piezometric data, although rarely available on new projects, is mentioned primarily because it is not infrequent for seepage analyses to be performed as a part of remedial measures for existing levees in which case piezometric data often are available.

a. Net head, H. The net head on a levee is the height of water on the riverside above the tailwater or natural ground surface on the landside of the levee. *H* is usually based on the net levee grade but is sometimes based on the design or project floodstage.

b. Thickness, Z, and vertical permeability, k_b, of top stratum. Where the thickness of the riverside blanket differs from that of the landside blanket, the designations, Z_{bR} and Z_{bL} are used. Similarly the permeability of the riverside and landside blankets are designated k_{bR} and k_{bL}, respectively.

(1) *Exploration*. The thickness of the top stratum, both riverward and landward of the levee, is extremely important in a seepage analysis. Exploration to determine this thickness usually consists of auger borings with samples taken at 3- to 5-ft intervals and at every

[1] References cited in this appendix are listed in Appendix A.

Table B-1
Examples of Transformation Procedure

Strata	Actual Thickness ft	Actual Permeability cm/sec	Transformed Thickness, ft $F_t = \frac{k_b}{k_n}$ for $k_b = 1 \times 10^{-4}$ cm/sec	
Clay	5	1×10^{-4}	1	5.0
Sandy Silt	8	2×10^{-4}	1/2	4.0
Silty Sand	5	10×10^{-4}	1/10	0.5
	Z=18			Z_b = 9.5

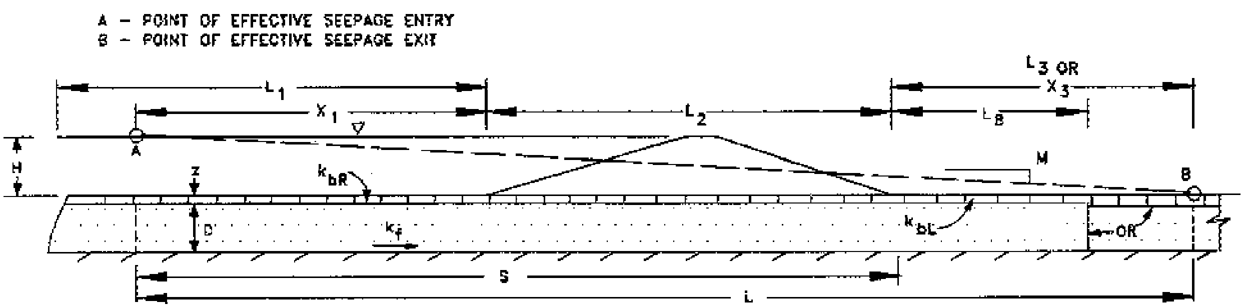


Figure B-1. Illustration of symbols used in Appendix B

change of material. Boring spacing will depend on the potential severity of the underseepage problem but should be laid out for sampling the basic geologic features with intermediate borings for check purposes. Landside borings should be sufficient to delineate any significant geological features as far as 500 ft away from the levee toe. The effect of ditches and borrow areas must be considered.

(2) *Transformation*. The top stratum in most areas is seldom composed of one uniform material but rather usually consists of several layers of different soils. If the in situ vertical permeability of each soil layer (k_n) is known, it is possible to transform the top stratum to an equivalent stratum of effective thickness and vertical permeability. However, a reasonably accurate seepage analysis can also be made by assuming a uniform vertical permeability for the top stratum equal to the permeability of the most impervious strata and then using the transformation factor given in Equation B-1 to determine a transformed thickness for the entire top stratum.

$$F_t = \frac{k_b}{k_n} \tag{B-1}$$

where F_t is the transformation factor. Some examples using this procedure are given in Table B-1 and in Figure B-1. A generalized top stratum having a uniform vertical permeability of 1 x 10^{-4} cm/sec and thickness of 9.5 ft would then be used in the seepage analysis for computation of effective blanket lengths. However, the thickness Z_{bL} may or may not be the effective thickness of the landside top stratum Z_t that should be used in determining the hydraulic gradient through the top stratum and the allowable pressure beneath the top stratum. The transformed thickness or the top stratum equals the in situ thicknesses of all strata above the base of the least pervious stratum plus the transformed thicknesses of the underlying more pervious top strata. Thus, Z_{bL} will equal Z_t only when the least pervious stratum is at the ground surface. Several examples of this transformation are given in Figure B-2. To make the final determination of the effective thicknesses and permeabilities of the top stratum, conditions at least 200 to 300 ft landward of the levee must be considered. In addition, certain averaging assumptions are almost always required where soil conditions are reasonably similar. Existing landward conditions or critical areas should be given considerable weight in arriving at such averages.

c. Thickness D and permeability k_f of pervious substratum. The thickness of the pervious substratum is defined as the thickness of the principal seepage-carrying stratum below the top stratum and above rock or other impervious base stratum. It is usually determined by means of deep borings although a combination of shallow borings and seismic or electrical resistivity surveys may also be employed. The thickness of any individual pervious strata within the principal seepage-carrying stratum must be obtained by deep borings. The average horizontal permeability k_f of the pervious substratum can be determined by means of a field pump test on a fully penetrating well as described in the main text. For areas where such correlations exist, their use will usually result in a more accurate permeability determination than that from laboratory permeability tests. In addition to the methods above, if the total amount of seepage passing beneath the levee (Q_s) and the hydraulic grade line beneath the levee (M) are known, k_f can be estimated from the equation

$$k_f = \frac{Q_s}{M} \tag{B-2}$$

d. Distance from riverside levee toe to river, L_1. This distance can usually be estimated from topographic maps.

e. Base width of levee and berm, L_2. The distance, L_2, can be determined from anticipated dimensions of new levees or by measurement in the case of existing levees.

f. Length of top stratum landward of levee toe, L_3. This distance can usually be determined from borings, topographic maps, and/or field reconnaissance. To determine this distance, careful consideration must be given to any geological feature that may affect the seepage analysis. Of special importance are deposits of impervious materials, such as clay plugs which can serve as seepage barriers. If the barrier is located near the landside toe, it could force the emergence of seepage at the near edge and have a pronounced effect on the seepage analysis.

g. Distance from landside levee toe to effective seepage exit, x_3. The effective seepage exit (Point B, Figure B-1) is defined as that point where a hypothetical open drainage face would result in the same hydrostatic pressure at the landside levee toe and would cause the

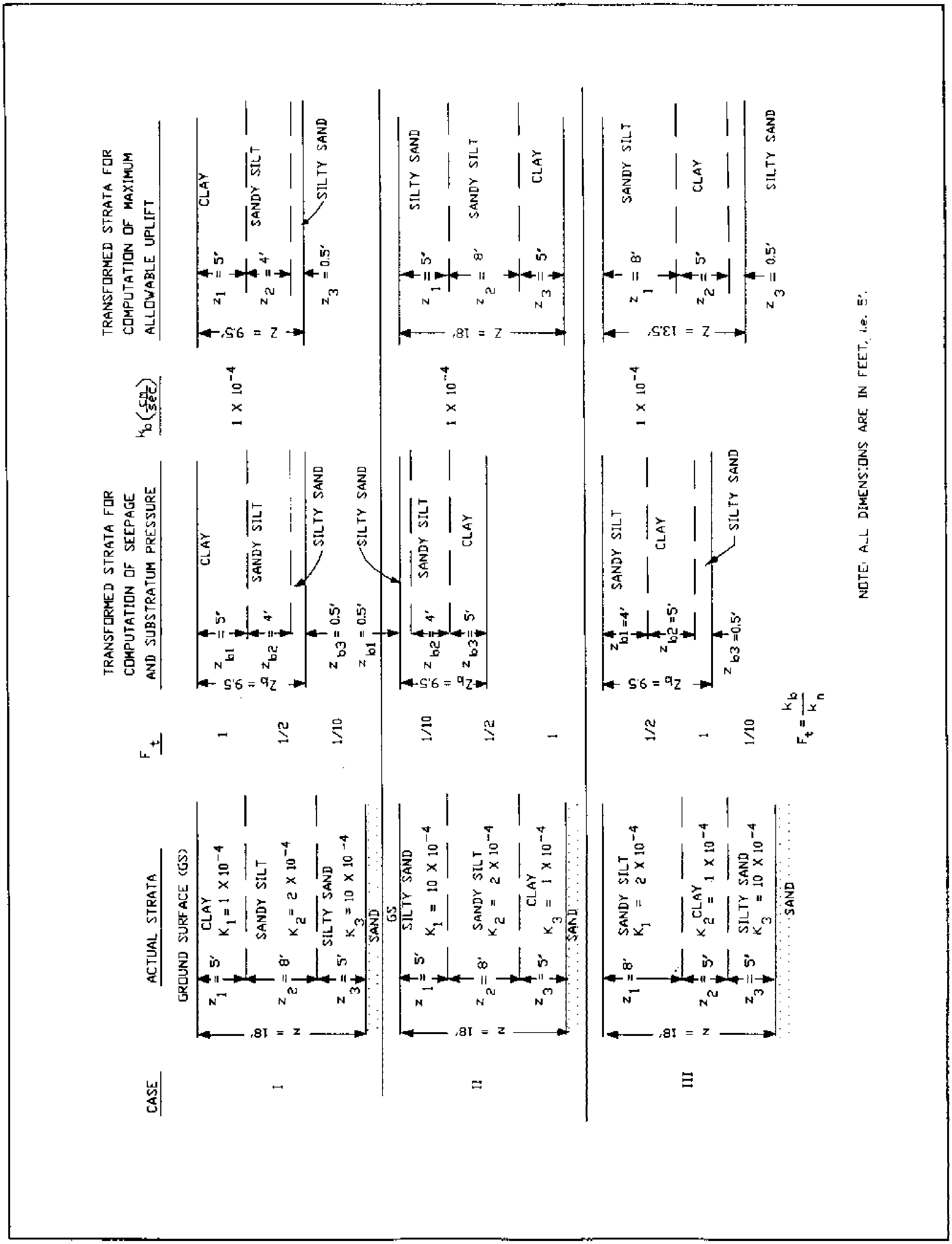


Figure B-2. Transformation of tops strata same amount of seepage to pass beneath the levee as would occur for actual conditions. Point B is located where the hydraulic grade line beneath the levee projected landward with a slope *M* intersects the ground water or tailwater. If the length of foundation and top stratum beyond the landside levee toe L_3 is known, x_3 can be estimated from the following equations:

(1) For $L_3 = \infty$

$$x_c = \frac{1}{C} = \sqrt{\frac{k_f \; Z_{bL} \; D}{k_{bL}}} \qquad \text{(B-3)}$$

where

$$C = \sqrt{\frac{k_{bL}}{k_f \; Z_{bL} \; D}} \qquad \text{(B-4)}$$

(2) For L_B = finite distance to a seepage block

$$x_3 = \frac{1}{c \; \tanh \; cL_b} \qquad \text{(B-5)}$$

(3) For L_3 = finite distance to an open seepage exit

$$x_3 = \frac{\tanh \; cL_3}{c} \qquad \text{(B-6)}$$

h. Distance from effective source seepage entry to riverside levee toe, x_1. The effective source of seepage entry into the pervious substratum (Point A, Figure B-1) is defined as that line riverward of the levee where a hypothetical open seepage entry face fully penetrates the pervious substratum. An impervious top stratum between the seepage entry and the levee would produce the same flow and hydrostatic pressure beneath and landward of the levee as would occur for the actual conditions riverward of the levee. Effective seepage entry is also defined as that line or point where the hydraulic grade line beneath the levee projected riverward with a slope, *M*, intersects the river stage.

(1) If the distance to the river from the riverside levee toe, L_1, is known, and no riverside borrow pits or seepage blocks exist, x_1 can be estimated from the following equation:

$$x_1 = \frac{\tanh \; cL_1}{c} \qquad \text{(B-7)}$$

where *C* is calculated from Equation B-4 using appropriate properties of the riverside top stratum.

(2) If a seepage block (usually a wide, thick deposit of clay) exists between the riverside levee toe and the river in order to prevent any seepage entrance into the pervious foundation beyond that point, x_1 can be estimated from the following equation:

$$x_1 = \frac{1}{c \; \tanh \; cL_1} \qquad \text{(B-8)}$$

where L_1 equals distance from riverside levee toe to seepage block and *c* is calculated from Equation B-4.

(3) The entrance conditions often are such that an assumption of a vertical entrance face is not reasonable. Two limiting cases are shown in Figure B-3. The additional effective length, ΔL_1, may be obtained for either Case A which assumes a uniformly sloping entrance face or Case B which assumes a combined infinite horizontal entrance face with a vertical entrance face, D', varying from *0* to *D* (see Figure B-3).

i. Critical gradient for landside top stratum, i_c. The critical gradient is defined as the gradient required to cause boils or heaving (flotation) of the landside top stratum and is taken as the ratio of the submerged or buoyant unit weight of soil, γ', comprising the top stratum and the unit weight of water, γ_w, or

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e} \qquad \text{(B-9)}$$

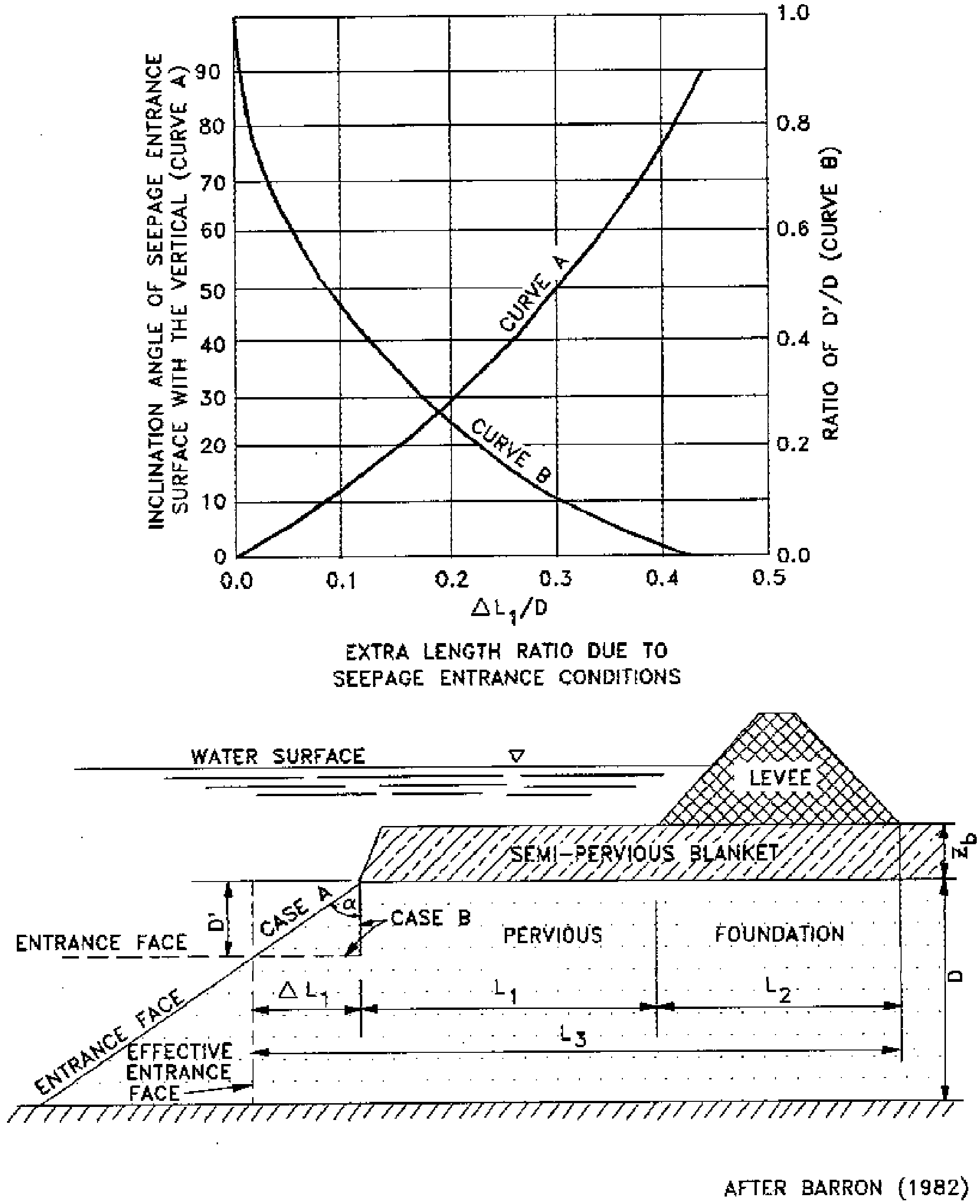


Figure B-3. Corrections for nonvertical entrance face (after Barron 1982)

where

G_s = specific gravity of soil solids

e = void ratio

j. Slope of hydraulic grade line beneath levee, M. The slope of the hydraulic grade line in the pervious substratum beneath a levee can best be determined from readings of piezometers located beneath the levee where the seepage flow lines are essentially horizontal and the equipotential lines vertical. The slope of the hydraulic grade line cannot be reliably determined, however, until the flow conditions have developed beneath the levee. If no piezometer readings are available, as in the case for new levee design, *M* must be determined by first establishing the effective seepage entrance and exit points and then connecting these points with a straight line, the slope of which is *M*.

B-5. Computation of Seepage Flow and Substratum Hydrostatic Pressures

a. General

(1) Seepage. For a levee underlain by a pervious foundation, the natural seepage per unit length of levee, Q_s, can be expressed by.

$$Q_s = \$ k_f D \qquad \text{(B-10)}$$

where $\$$ is the shape factor. This equation is valid provided the assumptions upon which Darcy's law is based are met. The mathematical expressions for the shape factor $\$$ (subsequently given in this appendix) depend upon the dimensions of the generalized cross section of the levee and foundation, the characteristics of the top stratum both riverward and landward of the levee, and the pervious substratum. Where the hydraulic grade line M is known from piezometer readings, the quantity of underseepage can be determined from

$$Q_s = M k_f D \qquad \text{(B-11)}$$

(2) Excess hydrostatic head beneath the landside top stratum. The excess hydrostatic head h_o beneath the top stratum at the landside levee toe is related to the net head on the levee, the dimensions of the levee and foundation, permeability of the foundation, and the character of the top stratum both riverward and landward of the levee. The head h_x beneath the top stratum at a distance *x* landward from the landside levee toe can be expressed as a function of the net head *H* and the distance *x*, although it is more conveniently related to the head h_o at the levee toe. When h_x is expressed in terms of h_o it depends only upon the type and thickness of the top stratum and pervious foundation landward of the levee; the ratio h_x/h_o is thus independent of riverward conditions. Expressions for $\$$, h_o and h_x for various boundary conditions are presented below.

b. Case 1 - no top stratum. Where a levee is founded directly on pervious materials and no top stratum exists either riverward or landward of the levee (Figure B-4a), the seepage Q_s can be obtained from Equation B-12. The excess hydrostatic head landward of the levee is zero and $h_o = h_x = 0$. The severity of such a condition in nature is governed by the exit gradient and seepage velocity that develop at the landside levee toe which can be estimated from a flow net compatible with the value of *S* computed from Equation B-12.

c. Case 2 - impervious top stratum both riverside and landside. This case is found in nature where the levee is founded on thick (15-ft) deposits of clay or silts with clay strata. For such a condition, little or no seepage can occur through the landside top stratum.

(1) If L_3 is infinite in landward extent or the pervious substratum is blocked landward of the levee, no seepage occurs beneath the levee and $Q_s = 0$. The head beneath the levee and the landside top stratum is equal to the net head on the levee at all points so that $H = h_o = h_x$.

(2) If an open seepage exit exists in the impervious top stratum at some distance L_3 from the landside toe (i.e., L_3 is not infinite) as shown in Figure B-4b, the distance from the feet toe of the levee to the effective seepage entry (river, borrow pit, etc.) is $L_1 = L_2$. The equation for the shape factor is given by Equation B-13, and the heads h_o and h_x can be computed from Equations B-14 and B-15, respectively.

d. Case 3 - impervious riverside top stratum and no landside top stratum. This case is shown in Figure B-4c. The condition may occur naturally or where extensive landside borrowing has taken place resulting in removal of all impervious material landward of the levee for a considerable distance. The shape factor is computed from Equation B-16. The excess head at the top of the sand landward of the levee is zero, and the danger from piping must be evaluated from the upward gradient obtained from a flow net.

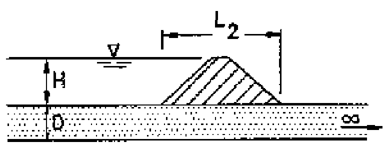
e. Case 4 - impervious landside top stratum and no riverside top stratum. This case is more common than Case 3 and occurs when extensive riverside borrowing has resulted in removal of the riverside impervious top stratum (Figure B-4d). For this condition, the shape factor is computed from Equation B-17; the heads h_o and h_x are computed from Equations B-18 and B-19, respectively.

f. Case 5 - semipervious riverside top stratum and no landside top stratum. This case is illustrated in Figure B-5a. The same equation for the shape factor as was used in Case 3 can be applied to this condition provided x_1 is substituted for L_1 in Equation B-16 resulting in Equation B-20. Since no landside top stratum exists, $h_o = h_x = 0$.

g. Case 6 - semipervious landside top stratum and no riverside top stratum. This case is illustrated in Figure B-5b. The shape factor is given by Equation B-21 and the heads h_o and h_x are computed from Equations B-22 and B-23, respectively.

h. Case 7 - semipervious top strata both riverside and landside. Where both the riverside and landside top strata exist and are semipervious (Figure B-6), the shape factor can be computed from Equation B-24. The head h_o is given by Equation B-25. The head h_x beneath the semipervious top stratum depends not only on the head h_o but also on conditions landward of the levee and can be computed from Equations B-26 through B-30.

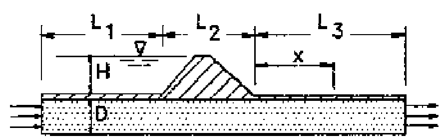
a. CASE 1 – NO TOP STRATUM



$$\mathcal{S} = \frac{D}{L_2 + 0.86D} \qquad \text{(B-12)}$$

$$h_o - h_x = 0$$

b. CASE 2 – IMPERVIOUS TOPSTRATUM BOTH RIVERSIDE AND LANDSIDE



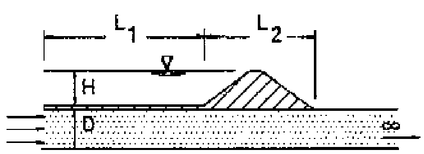
$$\mathcal{S} = \frac{D}{L_1 + L_2 + L_3} \qquad \text{(B-13)}$$

$$h_o = H\left(\frac{L_3}{L_1 + L_2 + L_3}\right) \qquad \text{(B-14)}$$

$$h_x = h_o\left(\frac{L_3 - x}{L_3}\right) \text{ for } x < L_3 \qquad \text{(B-15)}$$

$$h_x = 0 \text{ for } x > L_3$$

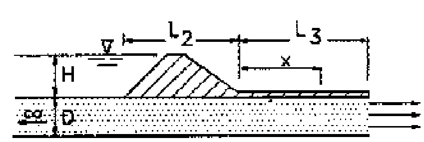
c. CASE 3 – IMPERVIOUS RIVERSIDE TOP STRATUM AND NO LANDSIDE TOP STRATUM



$$\mathcal{S} = \frac{D}{L_1 + L_2 + 0.43D} \qquad \text{(B-16)}$$

$$h_o = h_x = 0$$

d. CASE 4 – IMPERVIOUS LANDSIDE TOP STRATUM AND NO RIVERSIDE TOP STRATUM

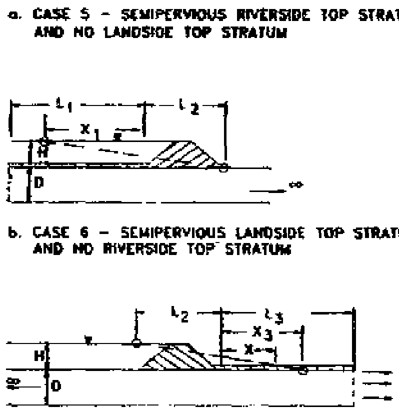


$$\mathcal{S} = \frac{D}{0.43D + L_2 + L_3} \qquad \text{(B-17)}$$

$$h_o = H\left(\frac{L_3}{0.43D + L_2 + L_3}\right) \qquad \text{(B-18)}$$

$$h_x = h_o\left(\frac{L_3 - x}{L_3}\right) \qquad \text{(B-19)}$$

Figure B-4. Equations for computation of underseepage flow and substratum pressures for Cases 1 through 4



$$\oint = \frac{D}{x_1 + L_2 + 0.43D} \qquad \text{(B-20)}$$

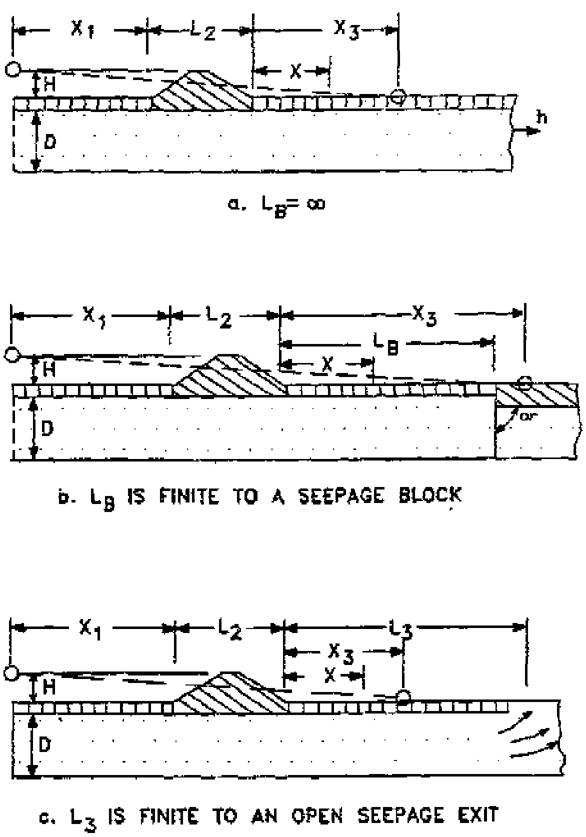
$$h_o = h_x = 0$$

$$\oint = \frac{D}{0.43D + L_2 + x_3} \qquad \text{(B-21)}$$

$$h_o = H\left(\frac{x_3}{0.43D + L_2 + x_3}\right) \qquad \text{(B-22)}$$

$$h_x = h_o\left(\frac{X_3 - x}{x_3}\right) \qquad \text{(B-23)}$$

Figure B-5. Equations for computation of underseepage flow and substratum pressures for Cases 5 and 6



$$\oint = \frac{D}{x_1 + L_2 + x_3} \qquad \text{(B-24)}$$

$$h_o = \frac{Hx_3}{x_1 + L_2 + x_3} \qquad \text{(B-25)}$$

$$c = \frac{k_{bL}}{k_f z_{bL} D}$$

$$h_x = h_o\, e^{-cx} \qquad \text{(B-26)}$$

$$h_x = h_o \frac{\cosh c\,(L_B - x)}{\cosh c\, L_B} \qquad \text{(B-27)}$$

$$h_x @\, L_3 = \frac{h_o}{\cosh c\, L_B} \qquad \text{(B-28)}$$

$$H_x = h_o \frac{\sinh c(L_3 - x)}{\sinh cL_3} \qquad \text{(B-29)}$$

$$H_x @\, L_3 = 0 \qquad \text{(B-30)}$$

Figure B-6. Equations for computation of underseepage and substratum pressures for Case 7